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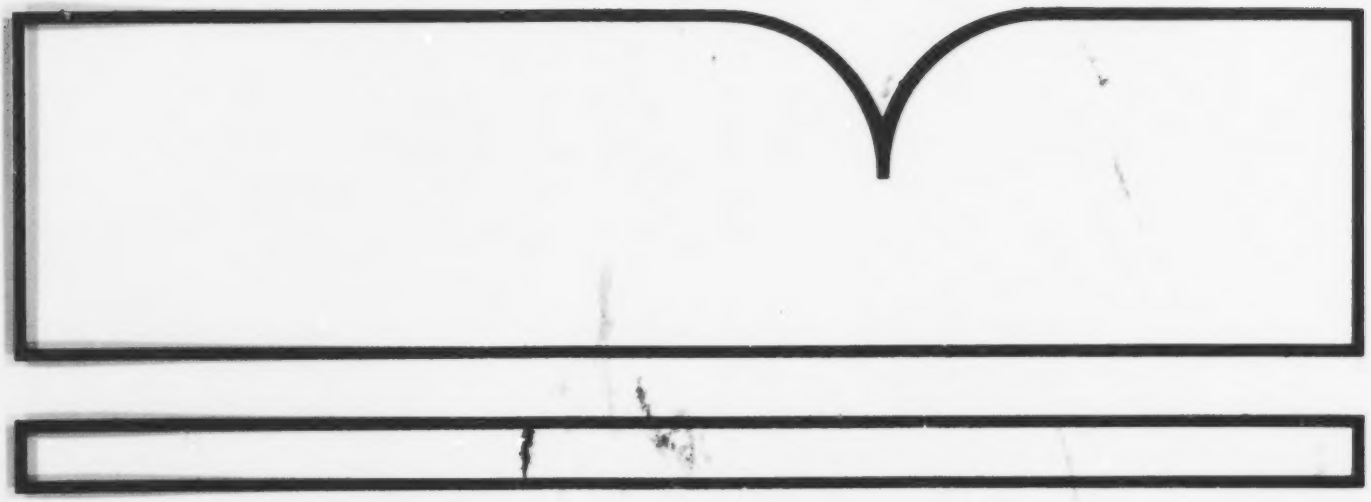

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16. ABSTRACT

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TEMPERATURES AND CURRENTS IN A STRATIFIED LAKE: A TWO-DIMENSIONAL ANALYSIS

Juan Heinrich, Wilbert Lick,[1] and John Paul[2]
University of Arizona,
Tucson, Arizona 85721

ABSTRACT. *Two-dimensional, time-dependent numerical models are used to predict the temperatures and currents in a stratified lake. In this study, (1) essential features of the observed distributions of temperatures and currents in large, stratified lakes, especially Lake Erie, are reproduced, (2) the effects of various governing parameters such as heat flux to the water, wind stresses, and turbulence are examined, and (3) the effects of changes in various parameters appearing in the turbulent eddy coefficients are investigated. The emphasis is on the general characteristics of thermocline formation, maintenance, and decay and therefore the time scales of interest are weeks and months rather than hours or even days.*

INTRODUCTION

The currents and the dispersion of contaminants in a non-stratified lake and even the numerical analysis and modeling of these processes are relatively well understood, at least in principle, even though the practical application of this knowledge leaves much to be desired. In contrast, the currents and the dispersion of contaminants in a stratified lake are not well understood, not even in principle.

Many important physical, chemical, and biological processes are significantly affected by stratification. Specific examples are the following: (1) the dispersion of contaminants throughout a lake; (2) the development of anoxia in a shallow stratified lake and the subsequent enhanced transfer of contaminants (such as phosphorous and trace metals) from the sediments to the overlying water; (3) the distribution of plankton; and (4) the resuspension of bottom sediments in a shallow lake, i.e., the effect of internal waves and hypolimnion currents on resuspension. Many other examples may be given.

Of course, considerable work has been done on understanding the basic dynamics of a stratified lake (e.g., see Hutchinson 1957, Mortimer 1974, and Watson 1977 for reviews of recent work). Extensive numerical analyses of the dynamics of non-stratified lakes (Simons 1972, 1974, 1979; Lick 1976; Sheng and Lick 1980; and Sheng *et al.* 1978) and preliminary work on the numerical analysis of flows where stratification is important (Paul and Lick 1974, 1981; Gedney, Lick, and Molls 1972, 1973; Bennett 1974, 1977, 1978; Vasseur, Funkquist, and Paul 1980) has been done. Three-dimensional models of flow in a stratified lake are presently being constructed and used. However, these three-dimensional models are inherently complex, are difficult to program and to use, and the results of any calculation are difficult to interpret.

In order to better understand transport in a stratified lake, we have developed and investigated simplified models. Our investigation began with a one-dimensional, time-dependent model. This one-dimensional model does not include convection but is still quite useful for understanding vertical diffusion (e.g., see Sundaram *et al.* 1970; Sundaram and Rehm 1973; Walters, Carey, and Winter 1978). A two-dimensional (one vertical and one horizontal coordinate), time-dependent model was then investigated. In this two-dimensional model, the vertical dynamics due to turbulent diffusion and convection are present as well as the horizontal dynamics due mainly to convection (Bennett 1974). The results of the two-dimensional model will generally be correct qualitatively, because of the effect of the inclusion

[1]University of California, Santa Barbara, California 93106
[2]Large Lakes Research Station, Grosse Ile, Michigan 48138

of the dominant physical processes, and quite often correct quantitatively. This two-dimensional model is the main subject of the present work.

The primary purposes of the investigation presented here were (1) to approximately reproduce the essential features of the observed distributions of temperatures and currents in large stratified lakes, especially Lake Erie, and (2) to investigate the effects of various governing parameters such as heat flux to the water, wind stresses and turbulence, and heat flux to the sediments. In doing the computations to accomplish these purposes, values for various parameters appearing in the eddy coefficients needed to be assigned. It is presumed that these parameters will have to be adjusted further when realistic three-dimensional calculations of Lake Erie are made and when other lakes of different geometry are studied. A secondary purpose of the present study was therefore to investigate the effects of changes in the various parameters appearing in the eddy coefficients. In the present analysis, the emphasis is on the general characteristics of thermocline formation, maintenance, and decay. Therefore, the time scales of interest are weeks and months rather than hours or even days.

A ONE-DIMENSIONAL, TIME-DEPENDENT ANALYSIS OF HEAT TRANSFER

In a lake, the transfer of heat is predominantly by convection and turbulent mixing in both the vertical and horizontal directions with additional, but usually smaller, contributions due to thermal radiation and molecular conduction. In the middle of a large shallow lake, the dominant heat transfer process is turbulent mixing in the vertical direction. As a first approximation to heat transfer throughout a lake and as an exceedingly good approximation to heat transfer in the middle of a large shallow lake, let us consider a one-dimensional, time-dependent, constant-area, vertical diffusion model of heat transfer. The basic equation governing the temperature T is then

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(K_v \frac{\partial T}{\partial z}\right) \tag{1}$$

where t is time, z is vertical distance (positive upwards with origin at the water-air interface), and K_v is the turbulent eddy diffusivity.

The eddy diffusivity is not constant but is a function of the heat transfer process itself, i.e., it is dependent on the thermal as well as the current structure in the lake. This dependence makes the above equation nonlinear, a nonlinearity which is essential to the formation, maintenance, and decay of the thermocline. The functional form of the eddy diffusivity is generally written as the product of the eddy diffusivity K_o in the absence of stratification and a stability function f which is dependent on the stratification, i.e.,

$$K_v = K_o\, f(\mathrm{Ri}) \tag{2}$$

where Ri is a Richardson number, is the ratio of buoyancy to inertia forces, and is defined by

$$\mathrm{Ri} = \frac{-\frac{g}{\rho}\frac{\partial \rho}{\partial z}}{\left(\frac{\partial \bar{u}}{\partial z}\right)^2} = \frac{g\alpha}{\rho\left(\frac{\partial \bar{u}}{\partial z}\right)^2}\frac{\partial T}{\partial z} \tag{3}$$

where $\bar{u}$ is the magnitude of the horizontal velocity at depth due to the combined action of waves (including internal waves) and currents, ρ is water density, g is the aceleration due to gravity, and α is the thermal expansion coefficient for water. The coefficient α is strongly dependent on the temperature and is given to sufficient accuracy by (Sundaram *et al.* 1970)

$$\alpha = 1.5 \times 10^{-5}(T-4) - 2.0 \times 10^{-7}(T-4)^2 \tag{4}$$

where the temperature T is in degrees Centigrade.

The quantity $\partial\bar{u}/\partial z$ presumably could be calculated by means of numerical models of waves and currents. This is a rather difficult task. It does not seem justified at present since the dependence of the eddy coefficients on the Richardson number is only approximate and is modified by the effects of the transport of turbulence (Launder and Spalding 1972; Marchuk *et al.* 1976, 1977). An alternative approach is as follows.

In the upper layers of a lake (and throughout a shallow lake), a major contribution to the turbulence is from surface wave action and this effect decays exponentially with depth. Because of this and because of the difficulties associated with calculating $\bar{u}$ directly, it is here assumed as a first approximation that $\bar{u}$ can be approximated as $u_o \exp(+z/D)$, where u_o is the maximum surface velocity, and D is an empirical constant of the same order of magnitude as the wave-lengtn of the surface waves. It follows that $\partial\bar{u}/\partial z$ can then be approximated as $(u_o/D)\exp(+z/D)$. The surface water velocity is related to the magnitude of the

wind stress τ_w by $\tau_w = \rho C_d u_o^2$ where C_d is a drag coefficient. By use of the above approximations and formulas, one obtains the following relation for the Richardson number

$$\mathrm{Ri} = \frac{g\alpha C_d D^2 e^{-2z/D}}{\tau_w} \frac{\partial T}{\partial z} \tag{5}$$

Forms of the Richardson number similar to that above have been considered and used by Pritchard (1960) and Kato and Phillips (1969).

Various forms of the stability function f(Ri) have been discussed in the literature (Munk and Anderson 1948, Lumley and Panofsky 1964, Okubo 1962, Kato and Phillips 1969, Sundaram *et al.* 1970). In the present calculations, the particular form of f(Ri) has been chosen to be

$$f = (1 + \sigma\ \mathrm{Ri})^{-3/2} \tag{6}$$

where σ is an empirical constant. From Eqs. (2), (5), and (6), it follows that K_v is given by

$$K_v = \frac{K_o}{\left(1 + \sigma_2 g\alpha \frac{D^2 e^{-2z/D}}{\tau_w} \frac{\partial T}{\partial z}\right)^{3/2}} \tag{7}$$

where $\sigma_2 = \sigma C_d$. Other similar forms for K_v have been investigated by us. However, the above form seems to give the most reasonable results by comparison with observations.

K_o, the eddy conductivity in the absence of stratification, unfortunately is not a constant but is a function of wind stress, depth of the lake, distance from the surface, and wave amplitude and fetch. On the basis of observations, numerical experiments, and the conventional wisdom, and as a first approximation, the functional dependence of K_o is taken to be

$$K_o = K_s(\tau_w, h)(c_1 + c_2 e^{+z/D}) \tag{8}$$

where c_1, c_2 are empircal constants and K_s is a function of both the wind stress τ_w and the lake depth h.

For Eq. (1), the boundary condition at the water-air interface, z = 0, is

$$q = -K_v \frac{\partial T}{\partial z} \tag{9}$$

where the heat flux q is specified. In general, this heat flux depends on the water temperature as well as conditions above the water such as wind speed, air temperature and humidity, and solar radiation (Edinger and Geyer 1965). For the present investigation, this dependence will not be considered and q will be assumed to be a known and simple function of time. At the sediment-water interface, z = -h, it can be assumed as a first approximation that there is no heat flux into the sediments. However, in shallow lakes such as Lake Erie, the heat flux into the sediments may be significant and in some circumstances must be considered. The appropriate boundary conditions are then that the heat flux and temperature are continuous across the sediment-water interface.

In general, the above equations must be integrated numerically. The resulting solutions give the temperature as a function of depth and time and, of course, give the location and time of formation of the thermocline. However, a simple expression for the approximate depth of the thermocline can be determined without recourse to numerical calculations as follows. The structure and formation of a thermocline are inherently dependent on the nonlinear nature of Eq. (1), where K_v is given by Eq. (7). this nonlinearity is due to the second term in the denominator of the expression for K_v. When the thermocline first forms, this latter term must be comparable in magnitude to the first term in the denominator, i.e.,

$$\sigma_2 g\alpha\ D^2 \frac{e^{-2z/D}}{\tau_w} \frac{\partial T}{\partial z} = 1 \tag{10}$$

It follows that the depth at which the thermocline forms is given by

$$z^* = -\frac{D}{2} \ln \left| \frac{\tau_w}{\sigma_2 g\alpha D^2} \frac{\partial T}{\partial z} \right| \tag{11}$$

or

$$z^* = -\frac{D}{2} \ln \left| \frac{\tau_w K_o}{\sigma_2 g\alpha D^2 q} \right| \tag{12}$$

where $\partial T/\partial z$ has been approximated by q/K_o.

It can be seen that as the wind stress τ_w and the eddy conductivity K_o increase, the thermocline depth increases, while as the thermal expansion coefficient α, the heat flux q, and the parameters σ_2 and D increase, the thermocline depth decreases. The above relationship is qualitatively correct and will be substantiated by the numerical calculations presented in the following.

In order to determine the dependence of the solution on the many different parameters appearing in the above formulation and to understand the general characteristics of thermocline formation, maintenance, and decay, numerous numerical experiments were performed. In these experiments, ranges of parameters were investigated but the emphasis was on those values of the parameters which were pertinent to large shallow lakes such as Lake Erie. For the central basin of Lake Erie, values of various parameters of interest are the following (Fish 1960, Burns and Ross 1972, Herdendorf 1980): the maximum depth is approximately 25 m, much of the basin is broad and flat with depths of 20 to 22 m, the thermocline typically forms at depths of 16 to 20 m with hypolimnion temperatures of 9 to 10°C slowly increasing through the summer, the thickness of the thermocline in midsummer is generally only a few meters (sometimes less than one meter), the heat flux to the water in the spring is approximately 0.003 cal/cm^2-sec slowly decreasing through the summer, and the average wind stress is on the order of 0.75 to 1.0 dynes/cm^2.

A series of calculations was made to investigate the dependence of the temperature field on the various parameters appearing in the above equations. A typical result is shown in Figure 1a. In this calculation, it was assumed that the depth was 22 m, the heat flux q was 0.003 cal/cm^2-sec, the wind stress τ was 0.75 dynes/cm^2, K_s = 15 cm^2/sec, σ_2 = 0.0045, and D = 900 cm. The initial temperatures were 6°C throughout. Thirteen nodal points were used in a forward-time, central-space algorithm.

It can be seen that the thermocline forms before 30 days (at approximately 20 days) and at a depth of 17 m. The hypolimnion temperatures at thermocline formation are slightly less than 9°C and slowly increase to 11°C. The epilimnion and hypolimnion are each well-mixed with a sharp temperature gradient dividing them.

In this calculation, a second thermocline formed (after 60 days) at a depth of 14 m. This phenomenon of multiple thermoclines was quite common in our calculations and is due to the variation of α with temperature. In the range of interest, $6°C < T < 20°C$, α increases by an order of magnitude. This significantly increases the nonlinearity of the expression for the eddy conductivity and has a major effect on the magnitude of the eddy conductivity. From Eq. (12), it can be seen that as the temperature and α increase, the thermocline depth should increase. However, instead of smooth changes in depth, the changes in depth occur in discrete jumps leading to multiple thermoclines. After the second thermocline has formed, the first thermocline will gradually decay in strength and will disappear. In order to determine the effect of grid size on the result, several calculations were performed with different grid sizes. From this it was determined that the location of the second thermocline is somewhat dependent on the grid size when a coarse grid is used, but the existence and location are not dependent on the grid size for a fine enough grid. Multiple thermoclines have been observed in lakes (Welch 1952, Hutchinson 1957, Torgersen *et al.* 1977) and it seems that the above mechanism may be partly responsible for this phenomenon.

The above calculation was made assuming a constant heat flux, a reasonable first approximation during the spring and early summer when the thermocline is first being formed. However, later in the summer and fall, changing heat flux obviously has a significant influence on the temperatures. A calculation with a variable heat flux is shown in Figure 1. For this case, it was assumed that the heat flux was constant for 50 days and then decreased at a linear rate such that it would be zero at 120 days. Note that no second thermocline has formed in this case. It will be seen later in the two-dimensional calculations that for constant heat flux a second thermocline is quite often formed but is generally eliminated when a decreasing heat flux is specified or when the wind is variable in direction.

Numerous other calculations were made (see Heinrich, Lick, and Paul 1981 for details). From these calculations, the following results were obtained: (1) increasing the heat flux causes the thermocline to form at a lesser depth, at an earlier time, and be stronger; (2) increasing the turbulence causes the thermocline to form deeper, at a later time, and be weaker; (3) heat transfer to the sediments in a shallow lake can have a significant effect on the temperatures in the hypolimnion (for the central basin of Lake Erie, a decrease of approximately 1°C); (4) effects of a sinusoidal 24-hour period, heat flux are limited to near-surface temperatures and do not affect the location and

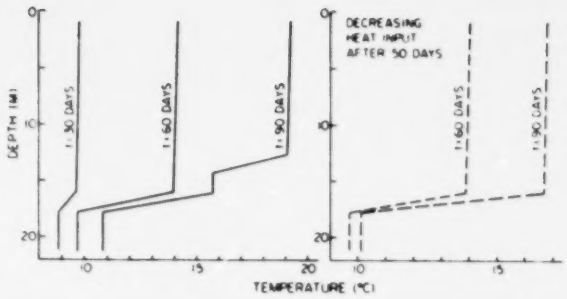

FIG. 1. Temperature profiles after 30, 60, and 90 days. -------- Constant heat flux; $q = 0.003$ cal/cm^2 sec. ——— Variable heat flux; constant heat flux to 50 days and decreasing after 50 days. Other parameters are: depth = 22 m, $K_s = 15$ cm^2/sec, $c_1 = c_2 = 1.0$, $D = 900$ cm, $\sigma_2 = 0.0045$, $\tau_w = 0.75$ dynes/cm^2.

time of formation of the thermocline; (5) the effects on the temperature distribution of thermal radiation (with a radiative absorption length for Lake Erie of 2.0 m as reported by Herdendorf (1980)) is also limited to near-surface temperatures. These results are all substantiated by our two-dimensional calculations.

BASIC EQUATIONS AND PARAMETERS FOR A TWO-DIMENSIONAL ANALYSIS

In the present, two-dimensional analysis, it is assumed that properties of the flow vary only in the vertical direction and one horizontal direction x but not in the other horizontal direction y. Coriolis forces are included since their effect on the flow in a large lake is significant. These forces induce velocities in the y-direction which are independent of y but are dependent on x and z. A net flux in the y-direction therefore results if a zero pressure gradient in the y-direction is assumed. Over sufficiently long time scales such that averaged seiche effects are negligible (and these are the time scales that we are interested in), this net flux in the y-direction must be zero. In the analysis, a pressure gradient in the y-direction independent of x and z is therefore assumed such that this net flux in the y-direction is zero. The basic model is a simplification to two dimensions of a three-dimensional, rigid-lid model previously developed and used (Paul and Lick 1974, 1981; Vasseur *et al.* 1980).

The basic equations used in the analysis are the usual hydrodynamic equations for conservation of mass, momentum, and energy plus an equation of state and are as follows:

$$\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) = 0 \tag{13}$$

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uw}{\partial z} - fv = -\frac{1}{\rho_r}\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(A_H \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial z}\left(A_v \frac{\partial u}{\partial z}\right) \tag{14}$$

$$\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial vw}{\partial z} + fu = -\frac{1}{\rho_r}\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left(A_H \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial z}\left(A_v \frac{\partial v}{\partial z}\right) \tag{15}$$

$$\frac{\partial p}{\partial z} = -\rho g \tag{16}$$

$$\frac{\partial T}{\partial t} + \frac{\partial uT}{\partial x} + \frac{\partial wT}{\partial z} = \frac{\partial}{\partial x}\left(K_H \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial z}\left(K_v \frac{\partial T}{\partial z}\right) \tag{17}$$

$$\rho = \rho(T) \tag{18}$$

where u, v, and w are the fluid velocities in the x, y, and z directions respectively, t is the time, f is the Coriolis parameter which is assumed constant, p is the pressure, ρ is the density, ρ_r is the ambient or reference density, A_H is the horizontal eddy viscosity while A_v is the vertical eddy viscosity, K_H is the horizontal eddy diffusivity while K_v is the vertical eddy diffusivity, g is the acceleration due to gravity, and T is the temperature.

In the present analysis, the horizontal eddy viscosity and diffusivity have been assumed constant. The vertical eddy diffusivity K_v is the same as in the one-dimensional analysis, i.e.,

$$K_v = \frac{K_s (c_1 + c_2 e^{+z/D})}{\left(1 + \sigma_2 \dfrac{g\alpha D^2 e^{-2z/D}}{\tau_w}\dfrac{\partial T}{\partial z}\right)^{3/2}} \tag{19}$$

where, as before,

$$K_s = K_s(\tau_w, h) \tag{20}$$

An analogous formula for the eddy viscosity is (Munk and Anderson 1948)

$$A_v = \frac{A_s (c_1 + c_2 e^{+z/D})}{\left(1 + \sigma_1 \frac{g\alpha D^2 e^{-2z/D}}{\tau_w} \frac{\partial T}{\partial z}\right)^{1/2}} \tag{21}$$

where A_s is assumed to be equal to K_s, and σ_1 is a constant of the same order of magnitude as σ_2. When the temperatures are calculated, it is sometimes found that the colder, more dense water is above warmer, less dense water. In this statically unstable situation, the unstable region is uniformly mixed (Bryan 1969).

At the water-air interface, $z = 0$, a heat flux and a stress due to the wind are specified,

$$-\rho K_v \frac{\partial T}{\partial z} = q, \tag{22}$$

$$\rho A_v \frac{\partial y}{\partial z} = \tau_x, \quad \rho A_v \frac{\partial v}{\partial z} = \tau_y \tag{23}$$

where τ_x, τ_y are the specified stresses in the x and y directions respectively. A rigid-lid condition, $w = 0$, is assumed. At the sediment-water interface, $z = -h$, where h is the depth, the conditions are those of no fluid motion and either zero heat flux or continuity of heat flux and temperature across the sediment-water interface.

CONSTANT-DEPTH BASINS

Wind Stress in the X-Direction

Let us first consider a constant-depth basin 100 km wide and 22 m deep, dimensions which approximate the central basin of Lake Erie. In the following calculations, it has been assumed that initially the temperatures are uniformly 6°C and the velocities are zero. A constant heat flux and a uniform wind stress in the x-direction are then imposed. After an initial rapidly changing transient, a quasi-steady state results in which the velocities and temperatures are changing relatively slowly.

For the first case presented, the parameters prescribed are: depth = 22 m, $q = 0.003$ cal/cm^2-sec, $\tau_x = 0.75$ dynes/cm^2, $\tau_y = 0$, $A_s = K_s = 15$ cm^2/sec, $c_1 = c_2 = 1$, $\sigma_1 = \sigma_2 = .00225$, $D = 900$ cm, and $A_H = K_H = 10^6$ cm^2/sec. Figure 2 shows the temperature distribution after a period of 30 days of constant heating and wind stress. Figures 3a, b, and c show the temperatures and velocities at 60 days.

Figure 2 shows that only a mild stratification is present after 30 days. It can be shown that the flow is slightly restricted by the stratification, but not significantly. At 60 days (Figure 3a), a strong thermocline has developed. The return flow (Figure 3b) is now more restricted to the epilimnion, and coastal currents (Figure 3c) are more developed. in the center of the basin, temperatures are almost identical to those predicted by a one-dimensional calculation indicating that the effects of convection and horizontal diffusion are not significant away from the vertical side boundaries.

Preliminary numerical experiments were also made to study the effect of changing wind directions. In these calculations, a constant wind stress was imposed for a few days until a quasi-steady state was reached. The wind direction was then reversed and, as expected, a transient resulted. This transient eventually decayed and a quasi-steady state was again reached. This latter state was generally quantitatively similar to that obtained without changes in the wind direction, i.e., the thermocline formed at the same depth and time and the temperatures were approximately the same. Because of this, and for simplicity, only constant imposed boundary conditions were considered in the next few examples. However, effects of wind reversal can be significant and examples of this are presented later.

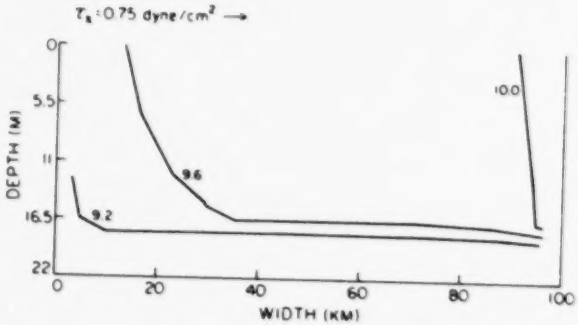

FIG. 2. Constant depth basin. Temperature distribution after 30 days. Other parameters are: depth = 22 m, $\tau_x = 0.75$ dynes/cm^2, $\tau_y = 0$, $q = 0.003$ cm^2/sec, $A_H = 10^6$ cm^2/sec, $K_H = 3 \times 10^5$ cm^2/sec, $K_s = A_s = 15$ cm^2/sec, $\sigma_1 = \sigma_2 = 0.00225$, $D = 900$ cm.

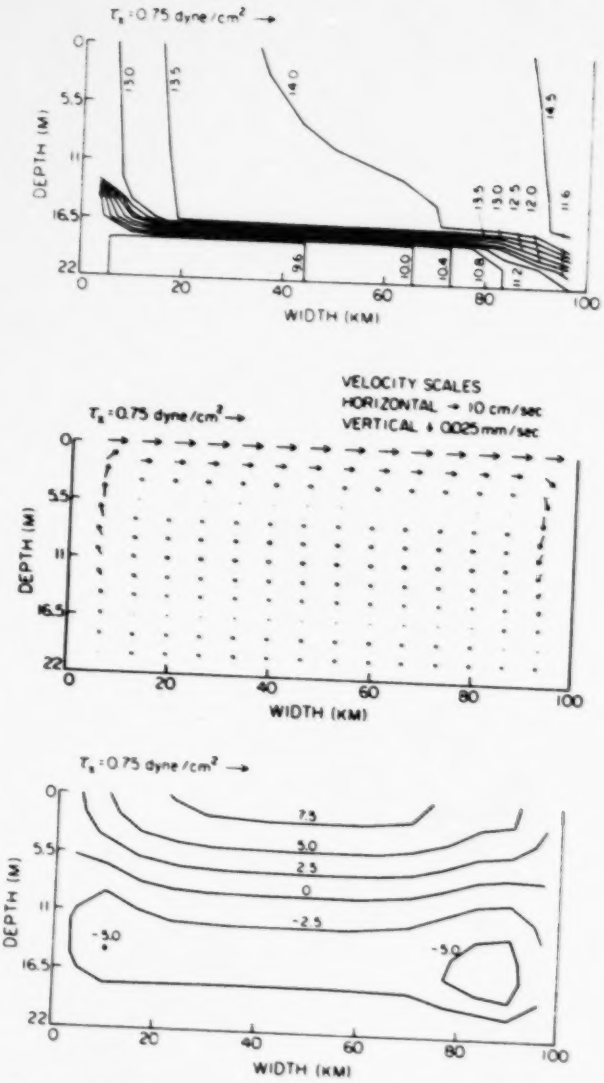

FIG. 3. Constant depth basin. (a) Temperature distribution, (b) Current velocities in the x-z plane, and (c) v velocities after 60 days. Same parameters as for Figure 2.

Wind Stress in the Y-Direction

In the above examples, it was assumed that the wind stress was in the x-direction, in the plane of the cross-section. A second set of calculations was made for the wind stress in the y-direction, perpendicular to the plane of the cross-section. The results are shown in Figures 4 and 5. By comparison of these figures with Figures 2 and 3, it can be seen

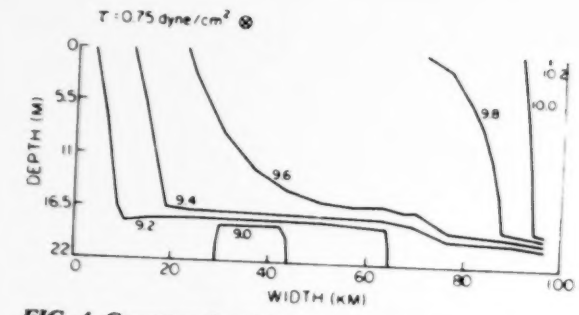

FIG. 4. Constant depth basin. Temperature distribution after 30 days. τ_y = 0.75 dynes/cm², τ_x = 0. Other parameters as in Figure 2.

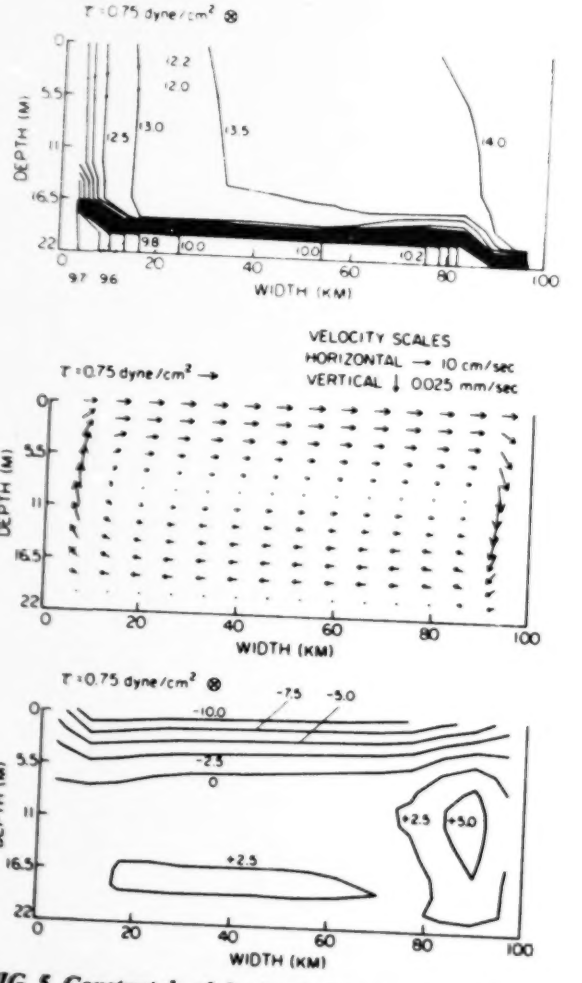

FIG. 5. Constant depth basin. (a) Temperature distribution, (b) Current velocities in the x-z plane, and (c) v velocities after 60 days. τ_y = 0.75 dynes/cm², τ_x = 0. Other parameters as in Figure 2.

that there are significant differences between the results for the two cases, not only for the currents but also for the temperature distributions.

For the wind stress in the x-direction, there is a strong near-surface flow in the direction of the wind and a broad return flow at depth. This return flow is primarily due to the Coriolis force and is a manifestation of the wind-driven Ekman layer. It may be recalled that the net (vertically integrated) flux for an Ekman layer (for a basin of infinite depth) is at 90° to the right of the wind stress and is therefore zero in the direction of the wind. In the present case, although the depth is finite, the wind-driven Ekman layer is essentially the same as that in a basin of infinite depth. Because of this, only a small pressure gradient in the x-direction is needed to return the flow from right to left.

For the wind stress in the y-direction, the net mass flux of the wind-driven Ekman layer is in the x-direction and is relatively large. A strong pressure gradient is therefore needed to return the surface flow. Because of this, upwelling and downwelling are much stronger and the return flow is significantly different from the previous case and is strongest just above the thermocline (compare Figures 3b and 5b). The temperatures are also quite different (compare Figures 3a and 5a). In the second example, the thermocline is displaced downward and will eventually disappear with time. However, when the wind direction was reversed approximately every 6 days, this did not occur and the thermocline was relatively stable at a depth of 16 to 18 m.

A VARIABLE-DEPTH BASIN

A series of calculations was made for a more realistic geometry corresponding to a section across the central basin of Lake Erie from Ashtabula, Ohio, to Port Stanley, Ontario (see Figure 6). The first calculation is for a wind stress in the x-direction. Results for the temperatures at 60 days are shown in Figure 6. The minimum hypolimnion temperature is about 10.8°C while the epilimnion temperatures are about 15°C. The thermocline is evident as is the (slight) warming of the waters in the shallow, nearshore areas. The vertical stratification is stronger in the center of the basin with more gradual stratification near shore. At later times (about 80 days), a second thermocline forms above the first thermocline in the upwelling region.

For a shallow basin such as Lake Erie, the turbulent eddies are restricted by the depth of the

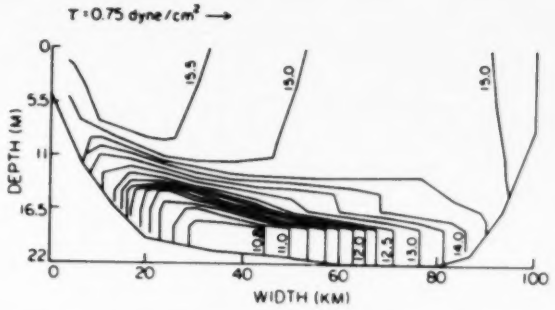

FIG. 6. Temperature distribution after 60 days for a variable depth basin of maximum depth 22 m with parameters: $A_H = K_H = 10^6$ cm²/sec, $A_s = K_s = 15$ cm²/sec, $c_1 = c_2 = 1$, $q = 0.003$ cal/cm²/sec, $\sigma_1 = 3\sigma_2$, $\sigma_2 = .003375$, $D = 900$ cm, $\tau_x = 0.75$ dynes/cm², $\tau_y = 0.0$ dynes/cm².

basin and therefore the eddy coefficients should decrease as the basin depth decreases. To investigate this effect, a calculation was made such that the eddy coefficients were proportional to the square of the depth, i.e.,

$$K_s = \left(\frac{h}{h_{max}}\right)^2 K_{s,max} \qquad (24)$$

This causes the thermocline to extend slightly more into the downwelling region on the right and less into the upwelling region on the left. The changes were slight. Nevertheless, all of the present calculations include this modification.

To investigate the effects of convection, a calculation was made with no wind stress. The temperature field for this case is shown in Figure 7. By comparison with Figure 6, the effects of downwelling and upwelling are apparent. The main effect is the displacement of the hypolimnion in the upwind direction. The temperatures in the center of the basin, both in the epilimnion and hypolimnion, are relatively unaffected by convection, at least for low to moderate winds as presecribed here. Strong winds of course can completely destroy the thermocline.

To investigate the effects of the horizontal diffusion of energy, a calculation was made with K_H decreased from 10^6 cm²/sec to 3×10^5cm²/sec (Figure 8). By comparison of Figures 6 and 8, a much stronger horizontal temperature variation is evident in the latter case, especially near shore. Also evident is that fact that the hypolimnion temperatures, even in the center of the basin, are

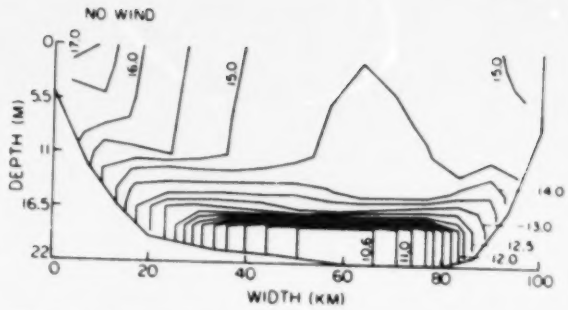

FIG. 7. Temperature distribution after 60 days for a variable depth basin with parameters: $A_H = K_H = 10^6$ cm²/sec, $A_s = K_s = 15$ cm²/sec (variable), $c_1 = c_2 = 1$, $q = 0.003$ cal/cm² sec, $\sigma_1 = 3\sigma_2$, $\sigma_2^2 = 0.003375$, $D = 900$ cm, $\tau_x = 0.0$ dyne/cm², $\tau_y = 0.0$ dyne/cm².

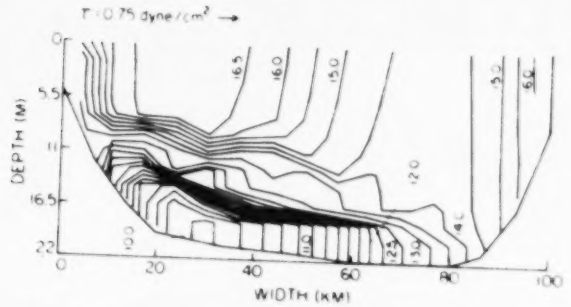

FIG. 8. Temperature distribution after 60 days for a variable depth basin with parameters: $A_H = 10^6$ cm²/sec, $K_H = 3x\ 10^5$ cm²/sec, $A_s = K_s = 15$ cm²/sec, $c_1 = c_2 = 1$, $q = 0.003$ cal/cm² sec, $\sigma_1 = 3\sigma_2$, $\sigma_2 = .003375$, $D = 900$ cm, $\tau_x = 0.75$ dynes/cm², $\tau_y = 0.0$ dyne/cm².

significantly affected. As indicated here, the horizontal diffusion of energy can be significant. By contrast, the horizontal diffusion of momentum is not significant except in narrow boundary layers near shore on the order of a few kilometers or less thick. In the horizontal momentum equations, the significant terms are the Coriolis and pressure gradient terms followed by the vertical diffusion term. In the energy equation, the first two terms are absent and vertical diffusion is now balanced by horizontal diffusion and convection.

The previous calculations were made in order to understand the general effects of various boundary conditions and parameters. A more realistic calculation was then made in order to model general and essential characteristics of stratified flow during the time of formation, maintenance, and decay of a thermocline. The calculation extended over a period of 180 days, from early spring to fall.

The following was assumed: (1) The heat flux was constant at 0.003 cal/cm²sec for 50 days and then decreased linearly so that it became zero at 120 days and negative thereafter, an approximation to real conditions; (2) The wind stress was constant at 0.75 dynes/cm² and directed to the right from 0 to 50 days, to the left from 50 to 70 days, to the right again from 70 to 130 days, to the left from 130 to 150 days, and then to the right from 150 to 180 days, an approximation to show and include effects due to varying wind direction. Other parameters were as follows: $A_s = K_s = 15$ cm²/sec, $D = 900$ cm, $A_H = 10^6$cm²/sec, $K_H = 3 \times 10^5$ cm²/sec, $\sigma_1 = \sigma_2 = 0.001875$, $c_1 = 0.5$, $c_2 = 1.5$, and no heat transfer to the bottom.

The results of the computations are shown in Figures 9 to 13 for times of 50, 60, 120, 150, and 180 days. Figure 9 shows the temperatures at 50 days. A

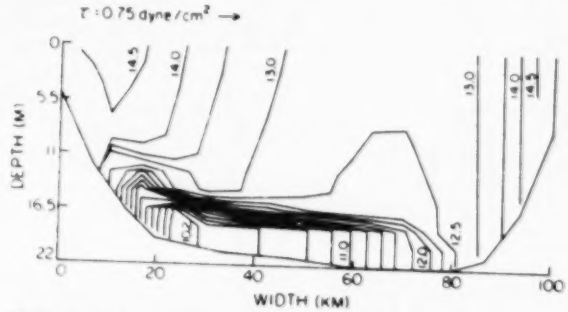

FIG. 9. Temperature distribution after 50 days for a variable depth basin with parameters: $A_H = 10^6$ cm²/sec, $K_H = 3 \times 10^5$ cm²/sec, $A_s = K_s = 15$ cm²/sec, $c_1 = 0.5$, $c_2 = 1.5$, $\sigma_1 = \sigma_2 = 0.001875$, $D = 900$ cm, $\tau_x = 0.75$ dynes/cm², $\tau_y = 0.0$ dynes/cm².

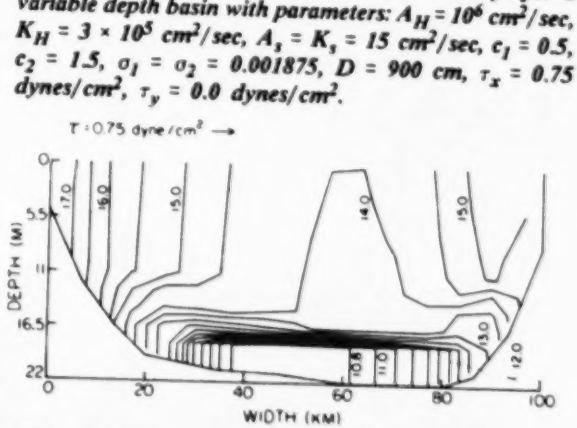

FIG. 10. Temperature distribution after 60 days for variable depth basin with heat flux decaying linearly and wind direction reversed after 50 days. Parameters as in Figure 9.

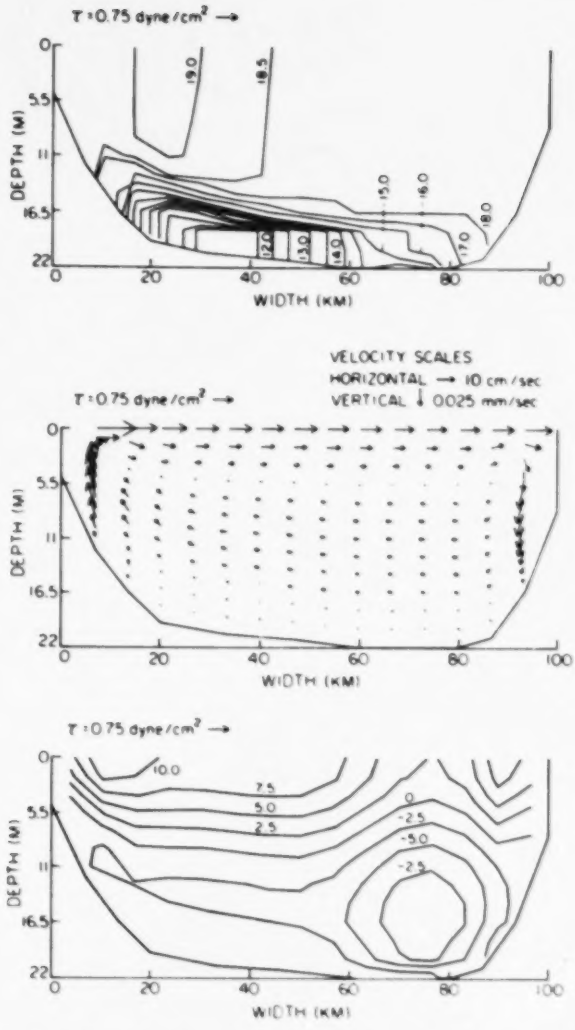

FIG. 11. (a) Temperature distribution, (b) Currents in the x-z plane, and (c) v velocities after 120 days for variable depth basin. Heat flux at this point is zero and becoming negative. Parameters as in Figure 9.

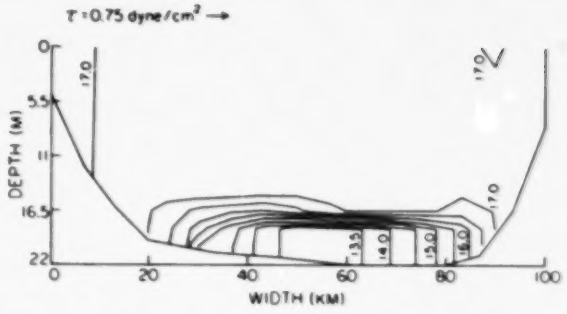

FIG. 12. Temperature distribution after 150 days for variable depth basin. Wind direction reversed from 130 to 150 days. Parameters as in Figure 9.

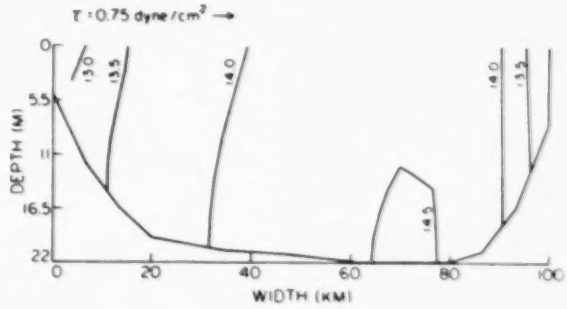

FIG. 13. Temperature distribution after 180 days for variable depth basin. Parameters as in Figure 9.

thermocline has formed at a depth of 16 to 18 m. The hypolimnion temperatures are 10 to 11°C. The epilimnion temperatures are 13°C in the middle of the lake and increase by 1 to 2°C in the near-shore areas. There is a broad return flow (primarily due to the wind-driven Ekman layer).

The results for the temperature at 60 days (10 days after wind reversal) are shown in Figure 10. The thermocline is quite flat. The epilimnion temperatures have increased by approximately 1.5°C while the hypolimnion temperatures have increased by less than 0.5°C.

There are two significant effects due to wind reversal. (1) Wind reversal causes a shift in the orientation of the thermocline so that the depth of the thermocline always increases in the downwind direction, i.e., the hypolimnion waters are shifted in the upwind direction. Although the location of the hypolimnion is significantly affected, the temperatures in the hypolimnion do not seem to be greatly affected by wind reversals. This motion of the hypolimnion is slow and 10 to 20 days are needed before a new quasi-steady state is reached. (2) In the above example, if the wind had not been reversed at 50 days, the temperatures and velocities at 60 days would be approximately the same as the case with a constant wind except as noted above and except that a second thermocline would have begun to form above the first thermocline in the upwelling region. This second thermocline would then continue to get stronger while the first thermocline would disappear. Wind reversal pre-

vents the formation of the second thermocline, at least in our calculations so far.

The wind again reversed at 70 days. The temperatures and velocities at 120 days are shown in Figures 11a, b, c. At 120 days, the epilimnion temperatures have increased to 19°C while the hypolimnion temperatures have only increased to 12 to 14°C. The flow is now more restricted to the epilimnion while the coastal currents have increased in strength. After 120 days, the heat flux is negative and cooling takes place. Cooling occurs more rapidly near shore eliminating the warm nearshore areas. At 150 days (Figure 12), the epilimnion is uniformly 17°C and the hypolimnion temperatures are 13 to 15°C. A thermocline exists but is quite weak and getting weaker. The more rapid cooling near shore can be observed in Figure 13 (180 days). Practically no vertical stratification exists and the flow is similar to that in a constant temperature basin.

Calculations were also made with the wind stress in the y-direction. As in the constant depth basin, significant differences are present between winds in the x- and y-directions. For the wind in the y-direction, a stronger and narrower return flow is located just above the thermocline. Upwelling and downwelling are also much stronger. The downwelling leads to a downward displacement of the thermocline, starting in the downwelling region near shore and eventually extending across the basin. Wind reversal counteracts this mechanism and stabilizes the thermocline.

SUMMARY AND CONCLUSIONS

By means of a two-dimensional, time-dependent model, a preliminary analysis of the temperatures and currents in a stratified lake has been made. Essential characteristics of the observed distributions of temperature and currents in basins comparable to the central basin of Lake Erie were reproduced. In particular, stratified flow during the time of formation, maintenance, and decay of a thermocline was investigated.

In general, significant differences were found between the two cases of wind stress parallel and perpendicular to the cross-section. These differences were due to the Coriolis force and resulted in a stronger, narrower return flow just above the thermocline when the wind stress was perpendicular to the cross-section. In addition, the effects of various governing parameters (such as heat flux, turbulence, and the direction of the wind stress) and parameters appearing in the eddy coefficients were investigated.

In the calculations presented, the eddy coefficients were assumed to satisfy Eqs. (19) and (21). Eq. (19) predicts that K_v in the hypolimnion (where the temperature gradients are relatively small) should be relatively large and approach $K_s(c_1 + c_2 \exp(+z/D))$ as $\partial T/\partial z \to 0$. This is consistent with the idea that the turbulence is generated at the bottom of the lake and/or by internal waves as well as at the air-water interface; and therefore K_v should be greater, perhaps much greater, than the minimum value of K_v in the thermocline. It could be argued, however, that if the turbulence is generated primarily at or near the air-water interface and is subsequently suppressed by the large temperature gradients at the thermocline, then K_v in the hypolimnion should be much smaller than K_v in the epilimnion and may be as small as the minimum value of K_v found in the thermocline. The real situation is not known and needs further research. Calculations were made where the value of K_v (and A_v) throughout the hypolimnion was taken to be the minimum value of K_v (and A_v) in the thermocline. In this case, the hypolimnion was not well mixed and the thermocline was not as strongly stratified. Numerous calculations were made in addition to those disucssed here and the report by Heinrich *et al.* (1981) should be consulted for these results and for additional details.

In general, the above calculations are in reasonable agreement with observations. Additional improvements need to be made to make the analysis quantitative. Parameters can be refined further so as to more closely approximate actual conditions. However, at this point, three-dimensional effects tend to become important and further improvements need to be made in conjunction with numerical results from three-dimensional calculations. In these latter calculations, it is necessary to use more realistic data as input to the analysis, and to verify the analysis by means of field data.

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